JASA

Journal of the American Statistical Association

SEPTEMBER 1975 • VOLUME 70
NUMBER 351 • PART II

JASA JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION
september 1975 volume 70 number 351, part II

Standards for Discussion and Presentation of Errors in Survey and Census Data . . . Maria Gonzalez, Jack L. Ogus, Gary Shapiro and Benjamin J. Tepping

EXAMPLES

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806 15th Street, N.W., Washington, D.C. 20005

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JASA JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

september 1975 volume 70 number 351, part II

ADVERTISING: Write to the Assistant Editor, University of Illinois, Chicago Circle, Box 6905, Chicago, Illinois 60680.

SUBSCRIPTIONS: Send to American Statistical Association, 806 15th Street, N.W., Washington, D.C. 20005.

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PUBLISHED: Quarterly by the American Statistical Association. Business Office: 806 15th Street, N.W., Washington, D.C. 20005. Printed by Lancaster Press, Inc., Lancaster, Pennsylvania. Editorial Office: University of Illinois, Chicago Circle, Box 6905, Chicago, Illinois 60680. Second class postage paid at Washington, D.C. and at additional mailing offices.

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Standards for Discussion and Presentation of Errors in Survey and Census Data

MARIA E. GONZALEZ, JACK L. OGUS, GARY SHAPIRO and BENJAMIN J. TEPPING*

INTRODUCTION

Each year the Bureau of the Census publishes a vast number of estimates on many subjects. It is the Bureau's responsibility to inform its data users of the important limitations of the estimates, both those due to sampling and those due to response and other nonsampling errors. The Bureau has developed standards for meeting this responsibility under varying conditions, depending on the type of report involved and on the nature and extent of the information available on the types of errors that affect the published estimates. These standards are contained in *Technical Paper 32*.[1]

Because of the basic importance to statistics of such standards and to make them available to a wider audience, this special supplement to the *Journal of the American Statistical Association* presents these standards for consideration in preparing statistical reports, based on the guidelines developed by the Bureau in its *Technical Paper 32*. The material presented here is essentially the same as that contained in *Technical Paper 32*, but rearranged to better suit the needs of the more general user and patron of survey data. In addition to presenting guidelines for consideration in presenting and interpreting survey results, this supplement discusses some relatively simple means of presenting error estimates for published data.

This supplement includes illustrations of alternative methods of presenting sampling and nonsampling errors that affect the data, some based on surveys carried out by the Bureau and others fictitious. The fact that a particular illustration uses a specific subject matter is not meant to imply applicability of the method to only that particular subject. Most examples are to be considered as excerpts from more complete texts and serve to illustrate only one or a few points.

It should be stressed that this report is not meant as a comprehensive guide to the derivation of error estimates in sample surveys. Rather, it is meant to suggest, on the one hand, the type of information about survey errors that should be included in reports containing survey data and to suggest to readers of such reports, on the other hand, the type of information about survey errors they should look for or expect to receive in evaluating the results of a survey.

We should like to thank the U.S. Bureau of the Census, particularly the Director, Vincent P. Barabba, for making this material available as a supplement to the *Journal of the American Statistical Association*.

ROBERT FERBER, Coordinating Editor

FOREWORD

In recent years, the actions of decision makers at all levels of Government have tended to encourage, if not to force, persons both inside and outside Government, who possess varying degrees of statistical understanding, to use Bureau of the Census products. The Bureau hopes, through the publication of this report as a special supplement to the *Journal of the American Statistical Association*, that this discussion on the presentation of errors in data from its surveys and censuses will be of use and value to the statistical community. The Bureau considers the present report as representing work in progress. Comments and suggestions for further improving communication between the Bureau's staff and users of data provided by the Bureau will be welcomed.

VINCENT P. BARABBA, Director
Bureau of the Census

* This guide is a revision of Technical Paper 32, *Standards for Discussion and Presentation of Errors in Data*, issued in March 1974 by the Bureau of the Census, Vincent P. Barabba, Director. Technical Paper 32 was the work of a committee whose members were Maria E. Gonzalez, Statistical Research Division; Jack L. Ogus, Industry Division; Gary Shapiro, Statistical Methods Division and Benjamin J. Tepping, Research Center for Measurement Methods, all with the U.S. Department of Commerce, Social and Economic Statistics Administration, Bureau of the Census, Washington, D.C. 20233. Thomas B. Jabine, former Chief of the Statistical Research Division, initiated the project and appointed the committee. He and many others of the Census Bureau staff contributed to the work by their critical comments and suggestions and by providing examples for inclusion in the report.

Robert Ferber, Coordinating Editor of the *Journal*, Maria E. Gonzalez and Gary Shapiro made the major revisions of Technical Paper 32 for this publication. The work of the Bureau of the Census staff members was carried out under the direction of Alva L. Finkner, Associate Director for Statistical Standards and Methodology; Harold Nisselson, Chief Mathematical and Statistical Advisor and Ralph H. Woodruff, former Chief of the Statistical Research Division.

[1] Technical Paper 32 is available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

Journal of the American Statistical Association
September 1975, Volume 70, Number 351, Part II

1. A POLICY ON ERROR INFORMATION

1.1 Definition and Interpretation of Sampling and Nonsampling Errors

All publications based on survey or census data should include appropriate statements that inform users that the data are subject to error arising from a variety of sources, e.g., sampling variability, response variability, response bias, nonresponse, imputation and processing error. Data furnished on computer tapes should be accompanied by such statements, which should include reference to the aspects of the design of the survey which affect the magnitude of errors from these various sources.

It is perhaps needless to note that the error categories used in a particular case should be defined and the concept of error explained in the course of this definition. The text of each report that presents sample data should include a statement that defines and interprets the term "sampling error." Nonsampling errors should also be discussed and the user made aware that the total error is larger than the estimated sampling errors shown. If possible, some quantitative information on nonsampling errors should be given, including the amount of imputation. The errors of published data should be prominently discussed in the introductory text of all comprehensive reports. This discussion should refer to both sampling and nonsampling errors, as shown in Example 1.

Example 1: The statistics in this report are estimates derived from a sample survey. There are two types of errors possible in an estimate based on a sample survey—sampling and nonsampling. Sampling errors occur because observations are made only on a sample, not on the entire population. Nonsampling errors can be attributed to many sources, e.g., inability to obtain information about all cases in the sample, definitional difficulties, differences in the interpretation of questions, inability or unwillingness to provide correct information on the part of respondents, mistakes in recording or coding the data obtained and other errors of collection, response, processing, coverage, and estimation for missing data. Nonsampling errors also occur in complete censuses.[2] The "accuracy" of a survey result is determined by the joint effects of sampling and nonsampling errors.

The particular sample used in this survey is one of a large number of all possible samples of the same size that could have been selected using the same sample design. Estimates derived from the different samples would differ from each other. The difference between a sample estimate and the average of all possible samples is called the sampling deviation. The standard or sampling error of a survey estimate is a measure of the variation among the estimates from all possible samples, and thus is a measure of the precision with which an estimate from a particular sample approximates the average result of all possible samples. The relative standard error is defined as the standard error of the estimate divided by the value being estimated.

As calculated for this report, the standard error also partially measures the effect of certain nonsampling errors but does not measure any systematic biases in the data. Bias is the difference, averaged over all possible samples, between the estimate and the desired value. Obviously, the accuracy of a survey result depends on both the sampling and nonsampling errors measured by the standard error and the bias and other types of nonsampling error not measured by the standard error.

An illustration of how to interpret sampling errors in terms of confidence intervals follows in Example 2.

Example 2: The sample estimate and an estimate of its standard error permit us to construct interval estimates with prescribed confidence that the interval includes the average result of all possible samples (for a given sampling rate).

To illustrate, if all possible samples were selected, each of these were surveyed under essentially the same conditions *and an estimate and its estimated standard error were calculated from each sample,* then:

i. Approximately 2/3 of the intervals from one standard error below the estimate to one standard error above the estimate would include the average value of all possible samples. We call an interval from one standard error below the estimate to one standard error above the estimate a 2/3 confidence interval.
ii. Approximately 9/10 of the intervals from 1.6 standard errors below the estimate to 1.6 standard errors above the estimate would include the average value of all possible samples. We call an interval from 1.6 standard errors below the estimate to 1.6 standard errors above the estimate a 90 percent confidence interval.
iii. Approximately 19/20 of the intervals from two standard errors below the estimate to two standard errors above the estimate would include the average value of all possible samples. We call an interval from two standard errors below the estimate to two standard errors above the estimate a 95 percent confidence interval.
iv. Almost all intervals from three standard errors below the sample estimate to three standard errors above the sample estimate would include the average value of all possible samples.

The average value of all possible samples may or may not be contained in any particular computed interval. But for a *particular* sample, one can say with specified confidence that the average of all possible samples is included in the constructed interval.

Examples 1 and 2 are offered as guides and are recommended for adoption as standard text. Appendix A, in the form of a fertility report that might be issued by the Bureau of the Census, is a more complete example of a discussion of data limitations. Individual authors may prefer to write other versions; if some other version is preferred, the author should take particular care that the concept of the confidence interval is used correctly. However, in all cases, the key idea of relating the sample estimates, their sampling errors and the average result of all possible repetitions of the survey should be observed. The term "true" value should not be used to refer to a value from a complete census, and illustrations of the use of standard errors should be included. The illustrations should be in terms of relative standard errors or absolute standard errors, depending on which are published.

The implications of the survey design for the various sources of error should be clearly indicated. The relevant

[2] A series of reports on the Evaluation and Research Program of the 1960 U.S. Censuses of Population and Housing, identified as Series ER 60, contains considerable information on the magnitudes of various types of nonsampling errors in the censuses.

aspects of the survey design include, but are not limited to, such things as the source of information (records or memory), use of a cutoff date, method of data collection, the character of the universe and of the frame, means of data reduction, etc. As noted in Example 1, the concept of total error and its relationship to sampling and nonsampling errors needs to be discussed, even though, as a rule, complete information on total error will not be available. Nevertheless, as much guidance as possible should be given, and where something specific has been done on nonsampling errors, the appropriate references should be provided. The effect of nonsampling errors on the accuracy of the estimates is discussed in Example 3.

Example 3.a: . . . the additional effect of the nonsampling error is smaller for the estimates of month-to-month relatives than for the estimates of monthly levels and is smaller for the estimates of total paint sales than for the estimates by the separate categories "industrial" and "trade."

Example 3.b: . . . it is believed to be minor for most general statistics estimates, somewhat greater for the product class estimates, and of greater importance for the estimates of assets, rental payments, voluntary labor costs and the value of inventories. For all items, the additional effect of the nonsampling errors is smaller for the estimates of year-to-year relatives than for the estimates of the annual totals.

Example 3.c: . . . since questions were obtained by direct interview, responses may have been affected by misunderstanding on the part of the enumerator. Since in small areas only one or two enumerators conducted all interviews whereas in larger areas a number of interviewers conducted interviews, there is a wider margin of relative error and response variability in data for small areas than for larger areas. The systematic field review early in the enumeration corrected some of the errors arising from misunderstandings by the enumerator.[3]

Example 3.d: . . . In the processing of data, careful efforts were made at each step to reduce the effects of errors. Errors occurred through failure to obtain complete and consistent information, incorrect recording of information on the schedules, incorrect transcriptions, etc.[4]

If problems of imperfect frames or errors in sample selection are known to be present, they should be described, as illustrated in Example 4.

Example 4: A sample of drycleaners and laundries was selected from two lists of drycleaning and laundry establishments, one consisting of members of the American Society for Dryer Dry-Cleaning and the other consisting of members of the National Institute of Laundries. No attempt was made to identify establishments that were included in both lists. Thus, such establishments have a greater chance of coming into the sample than those that are only on one list, since they may be selected from either of the two lists or both. Since no adjustment was made for this fact, all estimates in this report of the number of establishments in a particular class and of the gross and net income will tend to be too high. It is estimated that about 10 percent of the total number of establishments on the two lists are on both lists. The estimates of total income in the report will tend to be high by more than 10 percent, however, because in general the larger establishments will be the ones on both lists. For the same reason, estimates of average income per establishment will also tend to be high. In addition, a few very small establishments (about 3 percent of all establishments) are not on either list. This adds to the tendency for estimates of average income per establishment to be overstated. However, because of their small volume of business, the effect on estimates of total income is slight.

Comparisons between data sources may be affected by differences in procedures. Example 5 illustrates this point. A more extensive example is given in Appendix A.

Example 5: Some tables in this report give data from several sources, the *1970 Decennial Census*, the *Current Population Survey* and the *Health Interview Survey*. Particular care should be taken in comparing data from the different sources. Differences in procedures, phrasing of questions, interviewer training, etc., mean that the results from the several sources are not strictly comparable.

Any imputations that may have been made and their effect on the quality of the data should be discussed.

Example 6: The presentation of imputed data in a table and a description of the survey in the text are illustrated in Table 1.

Examples 1 and 2 which deal with nonsampling errors contain the minimum that should be said on this subject. Often, a fuller, more informative statement will be possible, and this part of the qualifying text should be enlarged accordingly, as illustrated by Examples 3 and 4 and Appendix A.

It should also be made clear that the different sources of error discussed do not take account of changes that may occur over time. If the survey data are to be used for forecasting or predictions, then there are problems relating to changes in the population and/or its characteristics that are not reflected in either sampling or nonsampling errors. Example 7 is an illustration of how this can be discussed.

Example 7: The estimates given in this report are for calendar year 1971. If these data are used for making estimates for other periods of time or for forecasting purposes, there are errors beyond the sampling and nonsampling errors previously discussed. This is because there are inevitably changes over time in the characteristics of people; and, in using estimates for other time periods, one either takes no account of these changes or does so imperfectly.

1.2 Presenting Information on Errors in Analytical Text Statements

Along with point estimates given in analytical text statements and press releases, information on the sampling and nonsampling errors of the estimates should be provided, where such errors affect the conclusions drawn.

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[3] See U.S. Department of Commerce, Bureau of the Census, *Effects of Interviewers and Crew Leaders*, Series ER 60, No. 7, Washington, D.C., 1968.

[4] See Minton, George, "Inspection and Correction Error in Data Processing," *Journal of the American Statistical Association*, 64, No. 328, (December 1969), 1256–75.

1. Yarn Consumed by Manmade Fiber Weaving Mills
(Thousands of pounds)

SIC code	*Type of yarn*	*Yarn consumed*	
		Second quarter 1970	*First quarter 1970*
	Yarn consumed, total	335,849	356,447
0228102	Cotton	30,419	32,569
0228110	Carded	23,915[a]	25,665[r]
0228120	Combed	6,504[a]	6,904
0228137	Rayon and acetate	86,305	95,723
0228130	Spun (100 percent)	27,833[a]	32,957
0282301	Filament (100 percent)	58,472	62,766
0282311	Acetate	40,993[a]	43,737
0282321	Rayon	17,479[a]	19,029
0282411	Nylon filament	14,713	15,321
0322933	Glass filament	32,668	33,299
0228013	All other yarns (including blends and mixtures)	171,744	179,535
0228143	Polyester (including content in yarn blends and mixtures)	78,866	80,203
0228104	Cotton content of blends and mixtures	32,016[a]	32,553[r]
0228135	Rayon and acetate content of blends and mixtures	36,658[a]	39,049
0228311	Wool, alpaca, and mohair yarn, and content in blends and mixtures	2,614	3,430
0228145	Acrylic fibers	5,220	5,183
0228146	Paper	c	c
0228142	Silk	611[b]	666
0228144	Saran and olefin	9,508	12,412
0224155	Rubber, elastic, lastex, etc.	153[b]	167
0228141	Nylon	3,011[b]	2,469
0228011	All other fiber yarn and fiber content of blends and mixtures.	3,087[c]	3,403[c]

[a] Six to seven percent of this item is imputed (See "Description of Survey" for a discussion of imputation rates).
[b] Eight to ten percent of this item is imputed (See "Description of Survey" for a discussion of imputation rates).
[c] Paper yarns are included with "all other yarns."
[r] Revised.
Source: Adapted from *Current Industrial Reports,* "Manmade Fiber Broadwoven Gray Goods," Second Quarter 1970.

Description of Survey: The statistics in this publication . . . were collected . . . from a panel of approximately 650 producers who account for about 95 percent of the total production. Estimates are included in the published figures for an additional 15 small producers reporting annually.

The current quarter's figures may also include estimates for respondents whose reports were not received in time for tabulation. Such missing figures are imputed from the quarter-to-quarter movements shown by reporting firms and are generally limited to a maximum of 5 percent to any one item. Individual items with higher imputation rates are footnoted.

The imputation rate is not an explicit indicator of the potential error in published figures due to nonresponse, because the actual quarterly movements for nonrespondents may or may not closely agree with the imputed movements. The probable range of difference between the actual and imputed figures is unknown. The degree of uncertainty regarding the accuracy of the data, however, increases as the percentage of imputation increases. Figures with high imputation rates, therefore, should be used with caution.

In addition, increased emphasis should be given to confidence intervals in lieu of point estimates. When feasible, text tables and graphs using confidence intervals should be published.

Unqualified point estimates in text statements in reports or press releases imply a false degree of exactness that encourages improper conclusions. Thus, point estimates that are quoted in the text should be suitably qualified. This is especially important in the rare case when it is appropriate to include in the text a figure that is not presented in the published tables. The source and reliability of the data should be noted. The following fictitious Example 8, typical of a standard short release by the Census Bureau, illustrates the incorporation of sampling error qualifications directly into the interpretative text.

Example 8: Prices of Z rocket containers were estimated to have increased by 0.8% and 0.6% in the second and third quarters, respectively, over the preceding quarters. The estimated increase between the third and fourth quarters was 0.5%. This indicates a −0.3% difference between the first estimated increase of 0.8% and the latest estimated increase of 0.5%. Allowing for the errors in the survey estimates, we can have 95-percent confidence that the difference falls in the range from −0.7% to 0.1%. Thus, though it is more likely than not that the rate of increase in the price of Z rocket containers has declined during this period, the evidence is not very conclusive.

The sampling error to which the estimates are subject may also be indicated parenthetically in the text, as indicated by Example 9.

Example 9: The average income for all poor families in 1967 was $1,076 (±1%) below the poverty threshold. The top 10% of all poor families had incomes averaging $86 (±5%) below the poverty level. The next 10% of the poor had family incomes $222 (±2%) below the poverty level. The poorest 10% had incomes averaging $3,086 (±3%) below the poverty standard.

The depth of poverty varied between white and all other families. To bring the incomes of the top decile of white poor families up to the poverty line would have required an addition of $62 (±20%), whereas to bring the incomes of the top decile of poor families of Negro and other races up to the poverty level would have required an addition of $138 (±7%) . . ..

The figures quoted above are the best estimates available from this survey, but they are imperfect measures (as is true of all types of estimates). For this reason, estimates are accompanied by their estimated relative standard errors, indicated parenthetically after the estimate. For example, the statement ". . . To bring the incomes of the top decile of white poor families up to the poverty line would have required an addition of $62 (±20%) . . ." means that the estimate of $62 is subject to a standard error of 20% or approximately $12.

The income data are subject to errors other than those due to sampling. These other errors would be present even if a census had been taken. For example, there is reluctance to reveal certain types of income, e.g., public assistance.

The sampling errors may also be indicated by footnotes or by a general statement. The latter is most appropriate when there is not a great deal of variation in the errors of the estimates quoted or when the sampling errors are unimportant with respect to the relationship discussed. Example 10 presents an illustration.

Example 10: The figures quoted here are estimates and, as such, are imperfect measures (as is true of all data). In general, small estimates of number of persons, percentage estimates where the bases are small, and estimated medians of small subpopulations tend to be relatively unreliable. However, in this report all comparisons are tested at a .05 risk of being rejected when true, and all interval estimates are made at the 95-percent confidence level.

Nonsampling errors that are known to be important, such as undercoverage of young black males in the decennial census or substantial imputation for nonresponse and for rejected data, should also be indicated. Thus, the following Example 11 of a portion of a technical report illustrates how discussion of limitations of the data other than sampling errors are built into such a presentation. This example also illustrates the parenthetical presentation of standard errors in the same units as the estimates to which they refer.

Example 11: The estimated net undercount of farms in 1964 is 401,000 (±32,000), or about 11.3% (±0.9) of the estimated total number of farms in the United States in 1964. This net undercount of 11.3% (±0.9) compares with 8.1% (±1.2) for 1959. From these data we can say with near certain confidence that the change in the net undercount was between −1.3 and 7.7%.

It seems that the apparent increase in the estimated net undercount from 1959 to 1964 resulted primarily from poorer coverage of small farms in 1964. The net undercount in the number of farms in 1964 was largest for "small" farms and decreased with increasing size of farm as measured in acres. This relationship was not observed in 1959. The estimated net undercount of acres in farms in 1964 of 6.1% (±1.2) cannot be said to differ from the corresponding figure of 6.0% (±0.9) for 1959.

Coming from a sample survey, the estimates and comparisons stated above are subject to sampling error. The estimated standard error is given parenthetically with each estimate. These can easily be converted into "confidence intervals." For example, the 95-percent confidence interval for the estimated 11.3% net undercount of farms is the range from 9.5% ($11.3 - 2 \times 0.9$) to 13.1% ($11.3+2 \times 0.9$).

The estimates of net error are also subject to nonsampling error, arising primarily from failure to locate and match census farms (correctly enumerated in the census) corresponding to some of the smaller coverage-check farms, which would lead to an overstatement of the net undercoverage. For a more detailed discussion of this and other sources of nonsampling error, see the Appendix[5] to this report.

Although statements in terms of confidence intervals are harder to write, they reduce the problem of interpreting the results when issues of statistical significance arise. Example 8 provides a good illustration of how the problem of interpretation of results can be reduced by use of confidence intervals. The practice of saying nothing when an observed difference is not "statistically significant" is not an adequate solution. It may reduce the attention given to important results, e.g., at turning points in a series, or it may encourage an interpretation of no change when, in fact, the band of uncertainty is large and an economically or sociologically important shift could have occurred. If all that can validly be said about an important figure is that the change lies within the range −0.2 to +0.3 percent, with the specified confidence of being correct, it would be desirable to say precisely that.

Such phrasing, additionally, would relax the arbitrariness of our choosing a particular multiple of the standard error to determine "significance." We may consider 2.0 times the standard errors as definitely significant. Others, taking account of their risks in using the data, may prefer higher or lower levels ranging from 1.0 to 3.0 standard errors.

The tabular presentation of confidence intervals rather than point estimates is especially desirable for text tables, which usually are small enough so that the additional space required would not cause a problem.

Example 12: Table 2 illustrates the presentation of estimates and corresponding estimated confidence intervals in the same table.

A means of presenting error ranges in the form of a graph is illustrated by Figures A and B (Example 13). These figures show how confidence intervals are presented within the framework of a bar diagram both for estimates of level and of differences in level, in this particular case with regard to unemployment. Such a

2. Unemployment in New York SMSA by Age, Sex and Color; 1969 Annual Averages
(Data based on a sample survey)

Age, sex and color	Unemployment			
	Level (000)	Confidence interval[a] (000)	Rate (%)	Confidence interval[a] (%)
Total	155	145–167	3.2	3.0–3.4
Men 20 yrs. and over	72	65–79	2.6	2.4–2.8
Women 20 yrs. and over	57	51–63	3.4	3.1–3.7
Both sexes, 16–19 yrs	26	22–30	9.4	8.1–10.7
White	124	115–133	3.0	2.8–3.2
Negro and other races	31	24–38	4.6	3.6–5.6

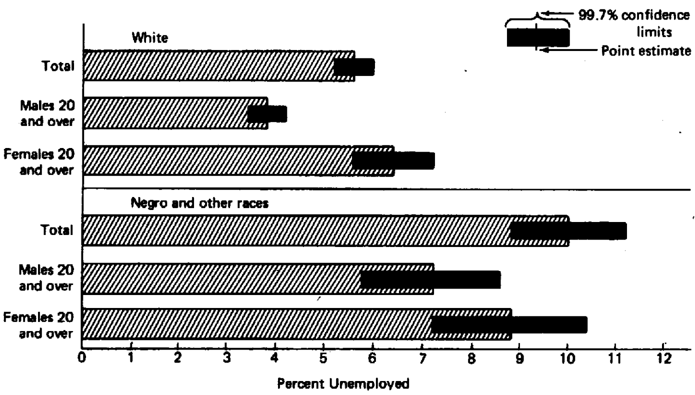
[a] There is 95 percent confidence that the interval includes the value being estimated.
Source: Adapted from Flaim, Paul O. and Schwab, Paul M., "Geographic Aspects of Unemployment in 1969," in *Employment and Earnings*, Vol. 16, No. 10, Bureau of Labor Statistics, (April 1970), 5–25.

[5] This appendix not included here—Ed.

figure is not easy to understand, and it is therefore desirable to accompany it with an explanation, as is done in Example 13.

Example 13: The graphic presentation of estimates and associated confidence intervals and interpretation of results are illustrated by Figures A and B and the following text.

A. Percent Unemployed by Race and Sex



B. Differences in Percent Unemployed by Race and Sex

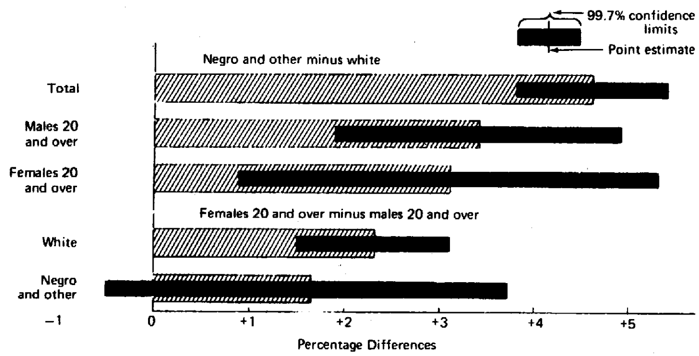


Figure A presents interval estimates of unemployment rates and Figure B, the difference in these rates, by race and sex. In Figure A, for each race-sex class the darkened horizontal bar covers a range which is nearly certain to include the estimate averaged over all possible repetitions of the sample. In Figure B, the darkened horizontal bars provide the same information for estimated differences between race-sex classes. For example, from Figure A it can be stated with nearly certain confidence that the unemployment rate for white males 20 and over lies between 3.4 and 4.2 percent. For males of Negro and other races 20 and over, the comparable range is 5.8 to 8.6 percent. Figure B provides more specific information about the estimated difference between these two classes. With nearly certain confidence it can be stated that the unemployment rate for males 20 or over of Negro and other races is 1.9 to 4.9 percentage points higher than for white males 20 and over.

It is of course not necessary to use a figure to present confidence intervals. Frequently, it is equally effective to present the standard errors in parentheses in a text table, with appropriate discussion of the effect of sampling error on interpretation of the estimates and with any cautions that may be needed for potential biases. One such means of doing so is illustrated by Example 14.

Example 14: We estimate that about 507,000 mink claws were shipped in May, including 419,000 common (black or brown) claws and 88,000 exotic (white) claws. The estimated percent change since the previous month (April 1970) and since the same month a year ago (May 1969) are shown in the following tabulation. The numbers in parentheses are the estimated standard errors, in percentage points.

ESTIMATED PERCENT CHANGE

Item	*From previous month*	*From previous year*
All claws	−4.3 (±3.4)	8.3 (±4.0)
Common claws	−3.9 (±3.4)	6.9 (±4.0)
Exotic claws	−6.4 (±3.4)	15.8 (±4.0)

In general, the sampling errors are too large to estimate the changes very precisely. However, there has clearly been a marked increase in shipments of exotic claws since last year. This may be the result of selective breeding for animals with white claws, a practice now becoming widespread in the industry. However, it should be noted that the classification of mink claws as common or exotic is subject to variation, for no industry-wide standard definition of exotic claws has yet been promulgated. The price of exotic claws has increased considerably in the last year, and the indicated increase in shipments of exotic claws may be to some degree the consequence of classifying light-colored claws as white.

1.3 Bases for Conclusions

Each analytical text, whether in a report or press release, should discuss the criteria applied to determine what conclusions may be drawn from the data. From any given body of data, a great variety of conclusions can conceivably be drawn. The first consideration in deciding what to discuss in a report is, of course, the substantive importance of a potential conclusion. But the conclusion can be drawn only if the data provided good evidence for it; the weight of the evidence usually depends on the reliability of the data. As an illustration, note that in the interpretation of the prices of rocket containers (Example 8) the possibility that prices of these containers may have increased is carefully qualified in the final sentence, making use of the information on sampling errors. Questions regarding simultaneous testing of multiple comparisons are not considered. All discussion is in the context of testing single comparisons. Moreover, in stating a conclusion, account has to be taken of possible sampling biases, as is shown in the case of the sample of drycleaners and laundries (Example 4).

Above all, the analyst has an obligation to the readers to indicate what considerations led to the conclusions that are drawn, as well as the considerations that prevented the drawing of other conclusions of substantive significance. Example 9 illustrates how such considerations are taken into account and how they are combined with the substantive findings, relating in this case to poverty levels.

1.4 Publication and Release of Unreliable Data

Estimates for individual cells of a published table should not be suppressed solely because they are subject to large sampling errors or large nonsampling variances, provided users are given adequate caution of the lack of reliability of the data. Protecting against misuse of data by suppressing them also prevents their valid use. On the other hand, data known to have very serious bias may be suppressed, as is shown by Example 21. Of course, this standard does not preclude the necessity for the exercise of good judgment in determining the level of detail to be presented in a given table.

Suppressing individual data cells in a given table because the sampling errors are large sometimes prevents users from compiling aggregates other than those shown. Moreover, the estimates that are subject to large relative sampling errors frequently are small, and the fact that the estimate is small is often sufficient information to be meaningful. For example, an estimate of four subject to a standard error of four could be very revealing if it indicated the number of doctors in a large rural county.

Similarly, tables that are not published at a given level of detail because most of the estimates would be highly unreliable in terms of sampling errors may be released to users with appropriate indications of the sampling errors. Data that have not been published because they are subject to serious biases require greater scrutiny before being released. If highly unreliable data are released, the recipient should be told in writing to include in any publication of the data statements specifying sampling errors and outlining any serious bias that is known to exist.

1.5 Frequency of Error Calculation

When a new, or largely new, sample is selected, sampling variances for a sufficient number of items should be estimated directly from the observed data to provide an adequate base for approximating the sampling errors of all the estimates to be published. Sampling errors should be computed for an adequate number of items from each group of items likely to exhibit different sampling behavior, e.g., groups which reflect different degrees of clustering or different types of crops in an agriculture survey. For single time surveys, sampling errors should be calculated for a sufficient number of items to give an adequate idea of the reliability of the survey estimates.

The relationships between variances for different statistics usually change slowly over time. Once these relationships have been determined, the standard error of many estimates can often be approximated satisfactorily from variances which are calculated directly from sample observations for other estimates. Such indirect methods have been used in the past and are clearly desirable when they are effective and appreciably reduce costs. Commonly, this will be the case for monthly and quarterly surveys and other surveys that yield a large number of estimates. However, a reasonable "capital investment" should be made in establishing a substantial base for approximating the standard errors whenever a new or substantially new sample is introduced. For repetitive surveys, the validity of the assumptions employed in approximating the current standard errors should be reevaluated occasionally, perhaps on a rotating basis. The variances and the measures needed to approximate indirectly derived standard errors should be recomputed at least once every two or three years for annual surveys, but more frequently for monthly and quarterly surveys.

1.6 Provision of Computer Tapes or Equivalent Copies of Data Records to Others

Responsibility to recipients of computer tapes or equivalent copies of data records parallels responsibility to users of one's publications. All records of data collected under Census law must be coded in such a manner that information about individuals cannot be identified, as in the case of the public use samples from the 1970 Census. In general, recipients should be cautioned about the existence of sampling errors and given some direction how to compute them from the data they obtain. They should be told that they, in turn, have a responsibility to present sampling errors in any compilations that they may publish. Users should also be given information about the magnitudes of the nonsampling variance, especially where tabulations are to be based on data collected from a small number of interviewers. Information available on biases should also be given to users. In turn, they should be told to include, in any reports they may publish, statements on the biases known to affect the data.

2. METHODS OF PRESENTING ERROR INFORMATION

2.1 Absolute Versus Relative Standard Errors

In reporting sampling variation, standard errors may be presented in either absolute or relative terms. Absolute standard errors can sometimes be related to the estimate more easily than relative standard errors, as illustrated by Example 11. This is especially apt to be the case for estimated percentages or rates as illustrated by Example 15, Table 3, relating to vacancy rates of different types of housing.

Example 15: Table 3 illustrates the presentation of estimates of percentages and their standard errors in adjacent columns of a table.

In some instances, however, the relative error may be more appropriate. Relative standard errors may require less space and may be applicable to a larger number of estimates in a given table, especially for aggregates that vary greatly in size or in their unit of measure.

Example 16: The presentation of estimates of level in a table and the corresponding average relative standard errors of level for each column of estimates is illustrated in Table 4.

3. *Vacancy Rates by Regions; Third Quarter 1968*
(In percentage points)

Region	Rental units occupied and for rent				Home-owner units occupied and for sale			
	Vacancy		Change in vacancy		Vacancy		Change in vacancy	
	Rate	Standard error of rate[a]	Change of rate	Standard error of change[a]	Rate	Standard error of rate[a]	Change of rate	Standard error of change[a]
United States	5.4	0.2	−0.3	0.2	1.1	0.1	+0.1	0.1
Northeast	3.4	0.3	−0.1	0.4	0.7	0.1	0.0	0.1
North Central	5.4	0.2	+0.6	0.4	0.9	0.1	+0.1	0.1
South	6.8	0.3	−0.4	0.5	1.4	0.1	+0.1	0.1
West	6.2	0.4	−1.4	0.6	1.4	0.1	+0.1	0.2

[a] See text for definition of standard error (Text not included here—Ed.).
Source: Adapted from U.S. Department of Commerce, Bureau of the Census, *Current Housing Reports: Housing Vacancies.* Series H-111, No. 54, 1969, 13.

In general, the preference between absolute and relative standard errors will vary depending on the nature of the table. The choice should be made by the authors in each individual case. In any event, it is important to avoid ambiguity in presenting standard errors, particularly in distinguishing between the absolute number of percentage points and the concept of relative error in percentage terms. For example, if the standard error of an estimated 38.8 percent is .4 percentage points, the relative standard error is approximately one percent. One means of making this distinction clear is to present separate tables for each case, as illustrated by Example 18.

2.2 What Multiple of Standard Error to Use

For consistency, all sampling errors in tables should be expressed at the same level. One standard error appears to be the most appropriate unit for this purpose. The user can then compute whatever multiple of the standard error is appropriate for the desired confidence interval.

While one standard error should be used consistently in tables, analysts writing interpretive text should choose what they consider to be the most appropriate multiple of the standard error and the corresponding confidence interval. There is not widespread agreement on whether a multiple corresponding to the 90-, 95- or 99-percent confidence interval would generally be most acceptable; for this reason, it is necessary to specify which confidence interval is being used in a particular case, as is illustrated by Example 10.

2.3 Placement of Error Information

Sampling errors should be presented as close as possible to the estimates to which they refer. Except where costs or operating problems make it unusually difficult, users

4. *Factory Sales of Paint, Varnish and Lacquer, 1968 to 1970*
(Thousands of dollars)

Month and year	Paint, varnish and lacquer total	Trade sales products			Industrial product finishes and special coatings		
		Total (2851010)	Paint and varnish (2851014)	Lacquer (2851500)	Total (2851067)	Paint and varnish (2851600)	Lacquer (2851700)
1970							
January	178,061	85,284	81,973	3,311	92,777	71,585	21,192
1969							
December	179,928	84,970	81,713	3,257	94,958	71,380	23,578
November	186,167	91,600	87,803	3,797	94,567	72,792	21,775
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1968							
December	175,726	83,042	79,905	3,137	92,684	74,202	18,482
November	196,888	92,693	89,136	3,557	104,195	82,810	21,385
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Average relative standard error[a] (percent)							
Monthly estimate	6	12	12	18	12	24	12

[a] See text for definition of relative standard errors (Text not included here—Ed.).
Source: Adapted from *Current Industrial Reports* M28F (20)-1, "Paint, Varnish and Lacquer," January 1970, 1.

should be alerted to the reliability of all the data by providing sampling errors, either in specific or general form, right in the table. Where the sampling errors cannot be shown in the same table as the estimates, the fact that the estimates are subject to sampling errors should be clearly indicated in each table, with a reference to where the sampling errors can be found.

It may be difficult to show sampling errors for all estimates in tables and may be less important in some cases than in others, but the exercise of skill and imagination can bring one a long way toward the goal of providing the user with immediately available information on the sampling errors of the data. The degree of detail will vary. For most users, a general idea of the reliability of the estimates should suffice, so that it would be adequate to present the sampling errors in broad categories that concisely cover all the estimates in a given table. One way of giving such a general idea without complicating the body of a table with too many figures is illustrated by Example 17, Table 5, relating to sawmill stocks of different woods in various parts of the country. Example 16, Table 4, relating to factory sales of paint, varnish and lacquer, illustrates how a general idea of the magnitude of the standard errors can be provided by adding only one additional line to a large table. Other illustrations of tabular presentation of sampling errors are given in Examples 12 and 21. An illustration of graphic presentation is given in Example 13.

Example 17: Table 5 illustrates the presentation of estimates marked with various symbols and notations in a table.

It is perhaps needless to note that such approximate estimates of reliability should not take the place of a more detailed discussion of reliability at a later point or perhaps in an appendix. Indeed, separate detailed sampling error tables should be published whenever only condensed information on the standard errors is incorporated in the data tables, since some users would want more complete and more precise information about sampling error than can be incorported in tables of estimates. This demand is presently met by the Bureau of the Census and other organizations in numerous publications by separate, comprehensive and detailed tables of sampling errors. This practice should be continued and extended where the costs of doing so are reasonable. Illustrations of the use of sampling error tables should also be included, as shown by Example 18.

5. Sawmill Stocks of Softwoods and Hardwoods by Census Geographic Divisions, End of Year: 1967 and 1966

(Millions of board feet, lumber tally)

Census geographic division	Total		Softwoods		Hardwoods	
	1967	1966	1967	1966	1967	1966
United States, total	5,138	5,215	3,879	4,114	1,259	1,101
East, total	2,393	2,226	1,161	1,149	1,232	1,077
New England and Middle Atlantic	**261	**233	(123)	(113)	(138)	(120)
East and West North Central	**197	**145	(75)	(69)	*122	*76
⋮	⋮	⋮	⋮	⋮	⋮	⋮

NOTE: All figures are sample estimates. Unless otherwise indicated, the figures are subject to relative standard errors of 5 percent or less. Figures marked with one asterisk have relative standard errors between 5 and 10 percent. Figures marked with two asterisks have relative standard errors between 10 and 15 percent. Figures in parenthesis have relative standard errors of 15 percent or more. For an explanation of the meaning and use of relative standard errors, see the note on sampling variations in Table 1 (Table 1 not included here—Ed.)

Source: Adapted from *Current Industrial Reports* MA24T(67)-1, "Lumber Production and Mill Stocks," 3.

6. Absolute and Relative Standard Errors of Estimated Number of Persons

Size of estimate	Standard error	
	Absolute	Relative (%)
25,000	700	3.0
50,000	1,100	2.1
100,000	1,500	1.5
250,000	2,300	.9
500,000	3,300	.7
1,000,000	4,700	.5
2,500,000	7,400	.3
5,000,000	10,000	.2
10,000,000	15,000	.2
25,000,000	23,000	.1
50,000,000	33,000	.1

The standard error of a percentage, computed by using sample estimates for the numerator and denominator of the percentage, depend upon the magnitude of the percentage as well as on the magnitude of the base of the percentage. The stub of Table 7 contains several levels of the estimated percentage. Note that the standard error of an estimated percentage is the same as the standard error of the complement of the percentage. The column headings of Table 7 give several possible magnitudes of the base of a percentage, ranging from 250,000 persons in the population to 50,000,000 persons in the population. For example, if the percentage of the labor force that is unemployed is 5 percent, and the number of persons in the labor force in the tabulation areas is 5,000,000, then the standard error of the unemployment rate is approximately 0.4 percentage points.

Example 18: Standard errors of estimated numbers of persons may be read from Table 6 and standard errors of estimated percentages may be read from Table 7. Table 6 gives both absolute and relative standard errors of estimates of the number of persons having any given characteristic covered by this report.

2.4 Supplementary Discussion on Errors

Frequently a small, specialized audience will be concerned with more details of the sampling and nonsampling errors for major surveys than can appropriately be covered in the general introductory text. A more complete discussion of the errors in major surveys should be prepared to satisfy the interests of this group. This material should be presented in a technical appendix, which may also include detailed tables of sampling

7. Standard Errors in Percentage Points of Estimated Percentages

Estimated percentage	Base of estimated percentage (thousands)			
	250	500	1,000	2,500
2 or 98	1.0	.7	.5	.3
5 or 95	1.5	1.1	.8	.5
10 or 90	2.1	1.5	1.0	.7
25 or 75	3.1	2.2	1.5	1.0
50	3.6	2.5	1.8	1.1
	5,000	10,000	25,000	50,000
2 or 98	.2	.1	.08	.06
5 or 95	.4	.3	.1	.08
10 or 90	.5	.4	.2	.1
25 or 75	.7	.5	.3	.2
50	.8	.6	.4	.3

Standard errors for values within the ranges of the tables may be approximated by interpolation. We give first an example of interpolation in Table 6. The estimate of the number of employees of a certain class is 66,000, but the table shows standard errors for 50,000 and 100,000. The standard error for 66,000 is approximately

$$1{,}100 + (1{,}500 - 1{,}100) \times \frac{66{,}000 - 50{,}000}{100{,}000 - 50{,}000} = 1{,}228$$

which we should round to 1,200. Interpolation in Table 7 may require interpolation for both the estimated percentage and the base for the estimated percentage. For example, the estimate for the proportion having a certain characteristic is 23.2 percent, on a base of 700,000. Interpolating between 10 percent and 25 percent for a base of 500,000, we obtain a standard error of 2.1 percentage points. Interpolating similarly for a base of 1,000,000 we obtain a standard error of 1.4 percentage points. Now interpolating between the bases of 500,000 and 1,000,000 we obtain a standard error of about 1.8 percentage points for a base of 700,000.

errors, or in a separate supplementary report,[6] so that casual users will not be unnecessarily burdened. Reasonably full descriptions of the survey methods used, including data processing procedures, should similarly be published, for the quality of data depends on the methods employed. These should be more extensive than the summary descriptions included in the regular survey releases but are not proposed as a substitute for them.

If a separate report is prepared, reference should be made to it in the regular releases. It may be desirable to present the more elaborate discussion of the survey errors and the survey methods together, because they are closely related and because the same group is likely to be interested in both. Among other things, such an appendix or supplementary report should contain a description of sampling error computation procedures, guidance for estimating standard errors for complex estimates and some discussion of seasonal adjustment, if seasonally adjusted data are published. It should also contain a discussion of the principal sources of nonsampling variance and bias in the particular survey (including the imputation procedure, Example 6, Table 1), as well as estimates of the magnitudes of their effects where those are available.

Such an appendix or supplementary report should also contain a discussion of the design effect on the reliability of the data. The square root of the "design effect" is defined as the ratio of the computed sampling error to the sampling error that would be obtained if a simple random sample and simple unbiased estimation were employed. The magnitudes of the design effects and why they vary for different characteristics should be discussed. Design effects are specific for each sample design and estimation procedure. Example 19 illustrates an abbreviated discussion of design effects for the *1969 Current Population Survey*.

Example 19: Table 8 records the $\sqrt{(\text{design effect})}$ for several items. These figures are ratios of the actual monthly Current Population Survey (CPS) sampling errors (using an annual average of the monthly data) divided by the sampling error appropriate for an unbiased estimate based on a simple random sample of persons, $\sqrt{(pq/n)}$. In general, one expects $\sqrt{(\text{design effect})}$ to be greater than 1.0 due to the sampling in CPS of clusters of households and of all persons within a household.

8. Design Effect for Current Population Survey Estimates, 1969

Characteristic	Percent of population 16+ possessing characteristic	$\sqrt{\text{Design effect}}$
Civilian Labor Force		
Total	60	1.15
Females	23	1.01
⋮		
Employed		
Negro and other races	6.2	.59
Self-employed	5.3	1.41
Unemployed		
Total	2.2	1.37
White females	.91	1.17
⋮		
Rural Residents		
Total nonfarm	25	10.62
Total farm	5.1	5.48

Source: Adapted from Banks, Martha J. and Shapiro, Gary M., "Variances of the Current Population Survey, Including Within- and Between-PSU Components and the Effect of the Different Stages of Estimation," *American Statistical Association Proceedings of the Social Statistics Section*, 1971.

In general, characteristics possessed by a relatively large percentage of the population are helped most by the ratio estimation used in CPS, which explains the relatively low $\sqrt{(\text{design effect})}$ for these characteristics in the table. Characteristics like "Negro and other races employed," though not a large percentage of the total population, represent a large percentage of certain age-sex-race groups for which separate ratio estimates are applied, and thus the low $\sqrt{(\text{design effect})}$ for this characteristic and other similar ones is not surprising.

[6] *The Current Population Survey: A Report on Methodology*, Technical Paper No. 7, is an example of such supplementary reports. In addition, the reports on the Evaluation and Research Program of the U.S. Censuses of Population and Housing in 1960, identified as Series ER 60, discuss various types of nonsampling errors in the censuses.

The highest $\sqrt{\text{(design effect)}}$ values found are for the rural characteristics. This is expected since clustering of persons within single households and between neighboring households is most extreme for such characteristics.

2.5 Charts for Standard Errors

In some instances, it is feasible to supplement detailed sampling error tables with charts which facilitate interpolation. This is illustrated by the example from the *1967 Health Interview Survey*[7] presented in Appendix B. As will be seen from that appendix, information on standard errors is given in table form for the more common values, while at the same time charts are provided to enable the reader to interpolate standard errors for other bases. Wider use of such devices, where applicable, is desirable.

A table should always accompany a chart because figures can be read from tables more easily. If feasible, the table intervals should be chosen so that linear interpolation between them would provide a good approximation.

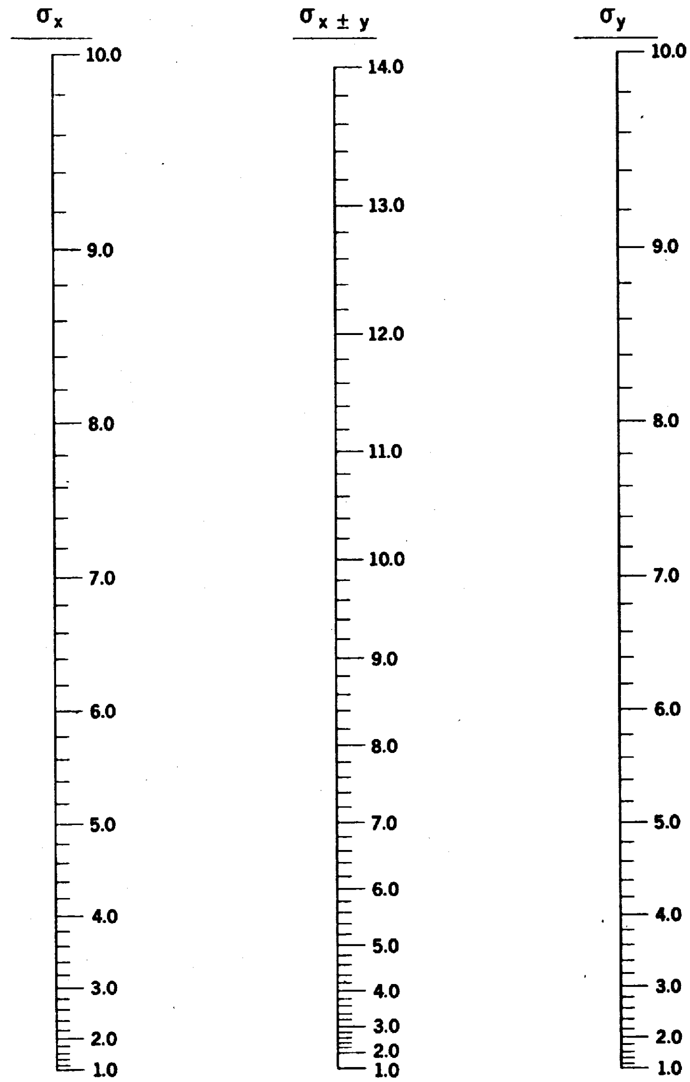
2.6 Approximating Sampling Errors for Derivative Statistics

Estimates of level are the foundation for almost all other estimates of interest, and nearly all survey data are published in this form. Ordinarily, therefore, the published sampling errors would refer to estimates of level. Frequently, however, estimates of change are recognized as being of prime interest to users. Consequently, sampling errors of estimates of change should be presented as prominently as feasible, as illustrated by Example 15. Alternatively, when this is not feasible, methods for approximating sampling errors of estimates of change from the sampling errors of level should be given.

One other means by which this may be done is to present a nomogram for estimating, say, the standard error of a sum or a difference of two independent estimates. This is illustrated by Figure C in Example 20. It should be stressed, however, that whenever such a figure is presented, its use should be explained both in general terms and by reference to a numerical example, as is done at the bottom of this particular nomogram.

Example 20: Figure C illustrates a method for graphically determining the standard error of a sum or difference of two independent random variables.

C. Nomogram: Standard Error of Sum or Difference
Independent Samples



Instructions for Use: If x and y are two independent estimates, then x + y and x − y are estimates of the sum and the difference, respectively. The standard errors may be approximated by the use of this nomogram. Locate the point on the σ_x scale that corresponds to the standard error of x, and the point on the σ_y scale that corresponds to the standard error of y. The scales may be read in any units (tenths, thousands, millions, etc.) provided that the *same* unit is used on all scales. Now connect the points by a straight line (a stretched thread is convenient) and read the value where the line crosses the $\sigma_{x\pm y}$ scale. This is the standard error of x + y and x − y. For example, suppose the standard error of x is 6,750 and the standard error of y is 4,700. A straight line between these values on the σ_x and σ_y scales crosses the $\sigma_{x\pm y}$ scale at about 8,250. An exact computation would have yielded the value 8,225.

When explicit estimates of change are published because of their dominant importance, their sampling errors may be presented in preference to the sampling errors of level. In such cases, methods of approximating the errors of level should also be given. This is illustrated by the lengthy footnote to Example 21, Table 9, which presents step-by-step instructions for computing relative standard errors of level from the published information in the table.

Example 21: Table 9 illustrates the presentation of estimates and their relative standard errors in the same table.

In addition to changes over time, users have interest in other derivative measures, e.g., ratios of totals for

[7] Adapted from U.S. Department of Health, Education, and Welfare, *Current Estimates from the Health Interview Survey: United States–1967*, National Center for Health Statistics, Series 10, Number 52, 35–36, 40.

9. General Statistics for Industry Groups and Selected Industries, 1958–1961[a]

Code	Industry group and industry	1961 Total employment (thousands)	...	1961 Value added by manufacture ($millions)	...	1961 Capital expenditures, new ($millions)	1960 Total employment (thousands)	1960 Value added by manufacture ($millions)	...	1959 Value added by manufacture ($millions)	Relative standard error (percent) of 1961, 1960 and 1959 estimates for columns[a]				
		A	...	F	...	I	J	K	...	O	A/J	F/K	I	K/O	O
–	All manufacturing establishments	16,341	...	164,292	...	9,264	16,763	164,003	...	161,315	1	1	1	1	1
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	Food and kindred products	1,704	...	20,195	...	1,033	1,717	19,692	...	18,614	1	1	2	1	1
201	Meat products	307	...	2,777	...	108	311	2,742	...	2,627	1	2	6	2	1
2011	Meat packing plants	190	...	1,914	...	73	194	1,912	...	1,834	1	2	9	2	2
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
202	Dairy products	281	...	3,248	...	181	288	3,200	...	3,041	1	1	4	1	1
2021	Creamery butter	16	...	151	...	(S)[b]	17	154	...	152	4	4	–	8	12
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
204	Grain mill products	117	...	2,075	...	127	118	1,998	...	1,895	1	1	2	2	2
2041	Flour and meal	27	...	448	...	24	27	444	...	411	2	1	9	5	7
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

[a] 1958 data based on the *Census of Manufactures*, all other data are sample estimates from the *Annual Survey of Manufactures*.
[b] (S): Withheld because estimate did not meet publication standards on the basis of a consistency review.
NOTE: This illustration does not include all the columns of the original table.
Source: Adapted from 1961 *Annual Survey of Manufactures Report*, 28–29, 46.

Footnote 6 to unabridged Table 9. General Statistics for Industry Groups and Selected Industries, 1958–1961: The relative standard errors shown for "1961 capital expenditures, new" (Column I), and "1959 value added by manufacture" (Column O), indicate the differences that can be expected between the estimated total shown and comparable complete-canvass results due to sampling fluctuations. Relative standard errors for the 1959 and 1960 employment figures and for other general statistics estimates are not included in this report (because of space limitations) but they are of the same general magnitude as the relative standard errors for value added.

The relative standard error columns which are headed A/J, F/K, and K/O indicate the differences that can be expected between the estimated year-to-year relatives for the specified years and comparable measures of change derived from complete-canvass results for "All employees, number" $\left(\frac{\text{1961 estimated total}}{\text{1960 estimated total}}\right)$, and for "Value added by manufacture": $\left(\frac{1961}{1960} \text{ and } \frac{1960}{1959}\right)$, respectively. While relative standard errors are not shown for the estimated 1960 and 1961 absolute totals for these statistics, they may be approximated as follows:

a. "1960 value added by manufacture:"
   Using columns "K" and "O", and relative standard error columns "K/O" and "O"
   (1) Multiply the estimate in column "K" by the relative standard error in column "K/O", and the estimate in column "O" by the relative standard error in column "O."
   (2) Square the two products obtained in step (1), and sum the squares.
   (3) Obtain the square root of this sum and divide the square root by the estimate in column "K".
b. "1961 value added by manufacture:"
   Using columns "F" and "K", relative standard error column "F/K", and the relative standard error derived in a. above for 1960 value added, repeat the procedure outlined in a., that is,
   (1) Multiply the estimate in column "F" by the relative standard error in column "F/K", and the estimate in column "K" by the relative standard error derived in a. for 1960 value added.
   (2) Square the two products obtained in step (1) and sum the squares.
   (3) Obtain the square root of this sum and divide the square root by the estimate in column "F".
c. "Total employment"
   A rough approximation to these relative standard errors is obtained by using the results obtained for "value added" in a. and b. above, since the relative standard errors of value added and employment statistics are generally of the same order. If more precise results are desired, reference should be made to the 1959–1960 *Annual Survey of Manufactures* volume, using standard error columns "A" and "J" to derive a standard error for the estimated 1960 total (as in a. above). This result may then be combined with column "A/J" (as in b. above) in the *1961 Annual Survey of Manufactures* volume to derive a relative standard error for the estimated 1961 total.

Note that the relative standard errors for the ratios are approximations. For a definition of and for an illustration of the use of relative standard errors see the section of the report "Source and Reliability of the Estimates."

one area to those for another, average unit prices (total value divided by total quantity) and temporal changes in derivative statistics such as productivity indexes. Practically, one cannot define all potential measures or develop and publish explicit sampling errors for them, but illustrative methods for computing at least rough approximations to the sampling errors of important and frequently used derivative statistics should be made available to the users of the tables. This is shown by Example 22 relating to a study estimating employment.

Example 22: The estimate of a certain characteristic for the North is 23.2% and the estimate of the same characteristic for the South is 20.0%. Since both of these estimates are based on samples, the estimated difference of 3.2 percentage points (23.2 – 20.0 = 3.2) is also subject to sampling error. If the base of both 23.2 and 20.0% were 700,000, interpolation in Table 7 of Example 18 shows that the estimated error of 23.2% is 1.8 percentage points, and the estimated standard error of 20.0% is 1.7 percentage points. If the two samples are independent, a rough estimate of the standard error of the difference may be obtained by multiplying the larger of the two standard errors by 1.4.[8] Thus, the standard error of the 3.2 percentage-point difference is approximately 2.5 percentage points. (This result could also be read from the nomogram shown in Example 20.) Based on these data, the two-thirds confidence interval is from 0.7% to 5.7%, and a conclusion that the average estimate of the percentage difference, derived from all possible samples, lies within a range computed in this way would be correct for roughly two-thirds of all possible samples. Similarly, a conclusion that the average estimate of the percentage difference, derived from all possible samples, lies within the interval from –1.8% to 8.2% would correspond to 95% confidence. Therefore, at the higher confidence level we would not conclude that the percent for the South is actually smaller than that for the North.

[8] The standard error of a difference is

$$\sigma_{A-B} = (\sigma^2_A + \sigma^2_B - 2\sigma_{AB})^{1/2}$$

where σ^2_A is the variance of A, σ^2_B is the variance of B, and σ_{AB} is the covariance of A and B.

Depending on the complexity of the problem and the space required for adequate discussion, methods for computing sampling errors of derivative statistics might be discussed separately, perhaps in a technical appendix. An example of such a discussion is given in Appendix C.

APPENDIX A: SAMPLING AND NONSAMPLING ERRORS WITH REFERENCE TO A PARTICULAR SURVEY

(Selections From a Hypothetical Fertility Report)

This appendix is an illustration of an extensive discussion of the limitations to which the data are subject: It refers to both sampling and nonsampling errors, as well as to other limitations of the data for various uses. This is the kind of discussion that would appropriately be included in a detailed report.

A.1 Sources of Data

The estimates are based on data obtained from the Censuses of the Population, Current Population Surveys, the Survey of Economic Opportunity, Vital Statistics Reports, unpublished data from the National Center for Health Statistics and several nongovernmental sources. A complete list of the references is given in the section, "References for Tables."[9]

Data for 1969 from the *Current Population Survey* are based on a sample spread over 449 areas comprising 863 counties and independent cities with coverage in each of the 50 States and the District of Columbia. Approximately 50,000 occupied households are eligible for interview each month. Of this number, 2,250 occupied units, on the average, are visited, but interviews are not obtained because the occupants are not found at home after repeated calls or are unavailable for some other reason. In addition to the 50,000, there

[9] This section not included here—Ed.

1'. Selected Age Data for Women in the Childbearing Ages: 1920, 1930, 1940, 1950, 1960, 1969[a]

Subject	Year					
	1969	*1960*	*1950*	*1940*	*1930*	*1920*
Women, 15 to 44 years (in thousands)	41,606	36,079	34,206	32,035	29,242	24,756
Women, 15 to 44 years, as a percent of total population	20.6	20.1	22.7	24.3	23.8	23.4
Percent distribution of women, 15 to 44 years, by age						
Total	100.0	100.0	100.0	100.0	100.0	100.0
15 to 19 years	22.0	18.3	15.5	19.2	19.8	19.2
20 to 24 years	19.6	15.3	17.2	18.4	18.9	19.2
25 to 29 years	16.1	15.3	18.3	17.6	17.0	18.4
30 to 34 years	13.7	16.9	17.2	16.1	15.6	15.9
35 to 39 years	13.7	17.7	16.7	15.0	15.5	14.9
40 to 44 years	14.9	16.4	15.0	13.6	13.2	12.4

Sources: For 1920–1950: *Census of Population*, 1950, Vol. I, part 1; for 1960: *Census of Population*, 1960, Vol. I, Part 1; for 1969: *Current Population Reports*, Series P-25, No. 441.
[a] The data in this table have no sampling errors.

Text of Table 1': During the past 50 years, the total population and the number of women in the childbearing ages have usually been increasing at different rates so that the proportion of the total population comprised of these women has fluctuated considerably. Between 1940 and 1960, the total population increased about 36 percent while the number of women in the childbearing ages increased only 13 percent, and as a result, dropped from 24.3 percent to 20.1 percent of the total population.

Changes in age structure among women in the 15 to 44 age group have also been pronounced. From 1920 to 1950, 35 percent to 38 percent were in the 20 to 29 age group in which birth rates are highest (Table 3'), but in 1960 the corresponding figure was down to 30.6 percent. By 1969, the figure had risen to 35.7 percent as the "baby boom" cohorts had begun to enter the prime childbearing ages.

The 32-percent increase in the crude birth rate between the late 1930s and late 1950s (Table 1') appears all the more striking because changes in age structure during the period were conducive to a sharp decline in the crude birth rate.

are also about 8,500 sample units in an average month which are visited but are found to be vacant or otherwise not to be interviewed.

For the 1965 data, the CPS sample was interviewed in 357 sample areas comprising 701 counties and independent cities. Approximately 35,000 occupied households were eligible for interview each month. Of this number, 1,500 occupied units, on the average, were visited, but interviews were not obtained because the occupants were not found at home after repeated calls or were unavailable for some other reason. In addition to the 35,000, there were also about 5,000 sample units in an average month which were found to be vacant or otherwise not to be enumerated.

The estimating procedure used in the *Current Population Survey* involves the inflation of the weighted sample results to independent estimates of the civilian population of the United States by age, race and sex. These independent estimates are based on statistics from the previous *Decennial Census of Population*; statistics of births, deaths,

2'. Estimated Number of Marriages and Median Age at First Marriage by Sex; Five-Year Averages, 1940 to 1969, and Single-Year Data, 1965 to 1970
(Standard errors in parentheses)

Year	Total marriages[a] (in thousands)	Median age at first marriage[b]	
		Men	Women
Five-year averages			
1965–1969	1,960[c]	23.0[d]	20.7[d]
1960–1964	1,605	22.8 (0.1)	20.4 (0.1)
1955–1959	1,516	22.6 (0.1)	20.2 (0.1)
1950–1954	1,567	22.9 (0.1)	20.3 (0.1)
1945–1949	1,857	23.2 (0.1)[e]	20.4 (0.1)[e]
1940–1944	1,619	24.3 (0.1)[f]	21.5 (0.1)[f]
Single-year data			
1970	2,179[c]	23.2 (0.1)	20.8 (0.1)
1969	2,146[c]	23.2 (0.1)	20.8 (0.1)
1968	2,069	23.1 (0.1)	20.8 (0.1)
1967	1,927	23.1 (0.1)	20.6 (0.1)
1966	1,857	22.8 (0.2)	20.5 (0.1)
1965	1,800	22.8 (0.2)	20.6 (0.1)

[a] At present, first marriages constitute about three-fourths of all marriages for both men and women.
[b] Medians are based on data from the *1940 Census* and the *Current Population Survey*. Medians based on marriage registration data would be slightly different.
[c] Data for 1969 and 1970 are provisional.
[d] Standard error estimated to be less than 0.05.
[e] Median is for 1947–1949.
[f] Median is for 1940.
Sources: For marriages, 1920–1967: *Vital Statistics of the United States*, Vol. 3, "Marriage and Divorce," 1967. For marriages, 1968: *Monthly Vital Statistics Report*, Vol. 19, No. 2, Supplement (2). For marriages, 1969–1970: *Monthly Vital Statistics Report*, Vol. 19, No. 12. For median age at first marriage: *Current Population Reports*, Series P-20, No. 212.

Text to Table 2': The annual number of marriages in the United States increased during the 1960s, and since 1968, the number has exceeded 2,000,000. While the number of marriages has historically been of some use in explaining trends in the number of births in succeeding years, this relationship can no longer be assumed because of the extent to which women now control both the quantity and timing of their fertility. From the late 1940s to the late 1950s, marriages decreased 18 percent, and births increased 22 percent. Between the late 1950s and late 1960s, marriages increased 29 percent while births decreased 15 percent.

Following the Second World War, the median age at first marriage dropped sharply for both men—from 24.3 to 23.2—and women—from 21.5 to 20.4. Between the late 1950's and 1970, the medians rose about one-half year—from 22.6 to 23.2 for men and from 20.2 to 20.8 for women. Each of these estimates is subject to a standard error of approximately 0.1 years.

3'. Median Age at First Marriage for Women by Year of Birth, Race and Educational Attainment: Birth Cohorts of 1910–1919, 1920–1929, 1930–1939 and 1940–1944
(Standard errors in parentheses)

Race and educational attainment	Birth cohort of women			
	1910–1919	1920–1929	1930–1939	1940–1944
White				
Total	22.0 (0.1)	20.9 (0.1)	20.1 (0.1)	20.2 (0.2)
Not a high school graduate	20.5 (0.2)	19.6 (0.2)	18.5 (0.2)	18.0 (0.3)
High school, 4 years	22.6 (0.2)	21.2 (0.2)	20.1 (0.2)	20.0 (0.2)
College, 1 year or more	24.0 (0.4)	22.5 (0.3)	21.9 (0.3)	21.4 (0.4)
Negro and other races				
Total	21.1 (0.3)	20.3 (0.3)	19.8 (0.3)	20.0 (0.5)
Not a high school graduate	20.9 (0.4)	19.4 (0.4)	18.8 (0.4)	18.8 (0.6)
High School, 4 years	21.4 (0.8)	21.2 (0.6)	20.9 (0.6)	20.1 (0.6)
College, 1 year or more	(NA)	(NA)	22.5 (0.8)	23.0 (1.2)

NOTE: NA: not available.
Source: *Current Population Reports*, Series P-20, No. 186.

Text to Table 3': Trends in median age at first marriage may be considered also on a cohort basis. Among white women the median age at first marriage dropped from 22.0 (0.1) for the 1910–1919 cohort to 20.1 (0.1) for the 1930–1939 cohort. There was a decline between the corresponding groups of women of Negro and other races; however, the sampling errors are sufficiently large that the magnitude of the decline is uncertain.

There is a positive relationship between educational attainment and median age at first marriage. The median age at first marriage among white women has typically been about 3 years higher for women with some college education than for women who did not complete high school. The differential is not so easily generalized among women of Negro and other races due to larger sampling errors.

immigration and emigration and statistics on the strength of the Armed Forces. To these figures were added the members of the Armed Forces living off post or with their families on post and the institutional population.

The *1967 Survey of Economic Opportunity* sample was interviewed in the same areas as the 1965 CPS sample, but the sample selected within these areas differed in size and composition. Approximately 29,000 occupied households were eligible for interview. Of this number, about 2,500 of the occupied units were visited, but interviews were not obtained because the occupants were not found at home after repeated calls or were unavailable for some other reason. In addition to the 29,000, there are also about 6,000 sample units which were visited but were found to be vacant or otherwise not to be interviewed. The sample was selected to have a disproportionately large sample of Negroes. The weights applied to each sample case were adjusted to reflect this. This sampling procedure results in more reliable estimates for Negroes but at the expense of reduced reliability for estimates for all races and for whites.

Vital statistics data are provided by the National Center for Health Statistics, Department of Health, Education, and Welfare. These data are drawn from a number of published and unpublished studies, some of which are based on samples and some on complete counts of vital statistics records. The data in Table 23[10] from the *Monthly Vital Statistics Report* are based on a 0.1 percent sample of birth records;

[10] Table 23 not included here—Ed.

4'. *Percent Single Among Women 15 to 44 Years Old by Age: 1920, 1930, 1940, 1950, 1960, 1969*
(Standard errors in parentheses)

Age of women	*Year* 1969	1960	1950	1940	1930	1920
Five-year age groups, 15 to 44 years						
15 to 19 years	88.8 (0.2)	83.9[a]	82.9[a]	88.1	86.8	87.0
20 to 24 years	35.7 (0.4)	28.4[a]	32.3[a]	47.2	46.0	45.6
25 to 29 years	10.7 (0.2)	10.5[a]	13.3[a]	22.8	21.7	23.0
30 to 34 years	6.1 (0.2)	6.9[a]	9.3[a]	14.7	13.2	14.9
35 to 39 years	5.3 (0.2)	6.1[a]	8.4[a]	11.2	10.4	11.4[b]
40 to 44 years	4.9 (0.2)	6.1[a]	8.3[a]	9.5	9.5	
Single years of age, 18 to 24 years						
18 years	83.5 (0.6)	75.6 (0.1)	75.4 (0.1)	82.3	80.0	80.0
19 years	69.4 (0.7)	59.7 (0.1)	62.4 (0.1)	73.0	70.2	70.3
20 years	55.9 (0.9)	46.0 (0.1)	49.8 (0.1)	62.8	60.5	60.0
21 years	42.8 (0.8)	36.6 (0.1)	39.9 (0.1)	54.4	52.9	52.5
22 years	32.6 (0.7)	25.6 (0.1)	30.4 (0.1)	46.2	45.0	44.9
23 years	23.9 (0.6)	19.4 (0.1)	24.0 (0.1)	38.7	38.1	38.3
24 years	18.5 (0.6)	15.7 (0.1)	19.6 (0.1)	32.9	32.4	33.0

[a] Standard error estimated to be less than 0.05.
[b] Percent single for 1920 data is available only for the combined category 35 to 44 years.
Sources: For 1920–1950: *Census of Population*, 1950, Vol. I, Part 1; for 1960: *Census of Population*, 1960, Vol. I, Part 1; for 1969: *Current Population Reports*, Series P-20, Nos. 187, 198 and 212. Also unpublished data from the Current Population Survey. Note that marital status is based on samples for 1950, 1960, and 1969.

Text to Table 4': Changes in the marital status of women in the childbearing ages may also be portrayed with the proportions single (never-married) by age. These proportions changed little between 1920 and 1940 and then declined sharply between 1940 and 1960. Between 1960 and 1969 the proportions single increased among women under 25. Since 1940 the frequency of spinsterhood among women 30 to 44 years old has declined sharply.

The data for women by single years of age show the changes in the proportions single in the ages at which most women first marry. In 1969 when the median age at first marriage was 20.8 (0.1), the interquartile range of ages was 18.9 (0.1) to 23.2 (0.1), meaning that one-half of all women first married in this age range.[11]

all other data from the *Monthly Vital Statistics Reports* are based on complete counts.

The appropriate Decennial Census publications provide descriptions of the sample designs used in the 1950 and 1960 Decennial Censuses. The data in Table 4' for the years 1950 and 1960 are from 20 and 25 percent samples of these Censuses. The 1960 Census data in Tables 12 through 17 are based on a 5 percent sample.[12]

A.2 Reliability of the Estimates

Estimates based on a sample may differ somewhat from the figures that would have been obtained if a complete census had been taken using the same schedules, instructions and enumerators. As in any survey work, the results are subject to errors of response and reporting as well as to sampling variability.

The standard error is primarily a measure of sampling variability; i.e., of the variations that occur by chance, because a sample rather than the whole of the population is surveyed. The standard errors on estimated percentages and estimated rates of children ever-born are inversely related to the size of the base, i.e., the smaller the size of the base, the larger the relative standard error.

The standard errors for data obtained from samples of the Decennial Censuses and samples of at least 20 percent of vital statistics records are considered unimportant in the evaluation employed in this report. All other sample estimates shown in the report have standard errors which were considered in the analyses. The conclusions stated in this report involving sample data are considered significant at the 95-percent confidence level. To test other findings which may be apparent in these tables, the standard errors should be considered. To find these standard errors, the reader should refer to the publications listed in the section, References for Tables.[13]

Data obtained from the Current Population Surveys, Vital Statistics Reports, Decennial Censuses and from nongovernmental sources are not entirely comparable. This is due in large part to differences in interviewer training and experience and in the differing survey processes. This is an additional component of error not reflected in the standard error tables.

A.3 Nonsampling Errors and Other Limitations

In this report, the term "Negro and other races" is used to describe persons of all races other than white. Data are shown for Negro and other races whenever data for Negroes alone are not available or are not available over the period of time shown.

Generally, statistics for the national population of Negro and other races reflect the condition of the Negro population, since about 92 percent of the population of Negro and other races is Negro.

In several tables, data are presented for the white and Negro populations but not for all races combined. Here, statistics for the white population tend to reflect the condition of the total population, since about 88 percent of the total population is white.

Vital statistics data on births are adjusted for under-registration through 1959 when it was estimated that the percent completeness of birth registration was 98.8 for the total population, 99.3 for whites and 96.2 for Negro and other races. Vital statistics data include Alaska beginning in 1959 and Hawaii beginning in 1960. The population bases

[11] Current Population Reports, Series P-20, No. 198, Table D.
[12] Tables 12–17 not included here—Ed.
[13] This section not included here—Ed.

used for the computation of annual birth rates are not adjusted for undercount. It is estimated that in the 1960 census, the enumeration for women 15 to 44 years old was 98.0 percent complete for all races, 98.8 percent complete for whites and 92.6 percent complete for Negro and other races.[14] Thus, the annual birth rates shown in this report are slightly higher than the rates that would have been obtained if corrected population bases had been used.

Among women ever married 14-years-old and over, in the 1960 census, six percent did not report on children ever born. Allocations for nonresponse were made by computer and were based on the responses of women who were similar to the nonreporting women in age and in other selected characteristics. In the *Current Population Survey,* in which nonresponse rates on children ever born are typically one to two percent, allocations are made in the same way. A content evaluation study of the 1960 census, in which data on children ever born for women not reporting in the census were obtained by reinterview methods, indicated that the allocation procedure worked well.

Never-married women are not asked about their fertility for reasons of public relations. Because of the social stigma attached to illegitimacy, some ever-married women may underreport births occurring before marriage, and some never-married women who have borne children may report themselves as presently or previously married. Comparison with vital statistics data indicates that the errors introduced by these two phenomena generally are small.

[14] U.S. Bureau of the Census, U.S. Census of Population: 1960, Vol. I, Part 1, Tables U and 46.

Another source of error results from the fact that the number of children ever born is sometimes given incorrectly, particularly by older women. A birth record check following the 1960 census indicated that the net error in the average number of children ever born for all ever-married women is small (overstatements and understatements are largely offsetting).

For a detailed discussion of nonsampling errors in data on children ever born, see Clyde V. Kiser, Wilson H. Grabill and Arthur A. Campbell, *Trends and Variations in Fertility in the United States*, Cambridge: Harvard University Press, (1968), Appendix B.

INTRODUCTION TO SECTION II

II. Age Structure and Marital Status

While period and cohort measures provide a comprehensive picture of levels and trends in fertility, they do not give explicit recognition to two important population characteristics, i.e., age structure and marital status. Among women in the childbearing ages, birth rates vary with age of woman, and birth rates at each age are much greater for married than for unmarried women. Tables 1′–4′ present data on age structure and marital status pertinent to an analysis of fertility, while the accompanying texts mention some cases in which changes in these characteristics have influenced trends in fertility.

In the accompanying texts, the figures in parentheses are estimates of the standard errors of the associated estimates. For a general discussion of sampling and nonsampling errors, see Sections A.2 and A.3.

APPENDIX B: GUIDE TO USE OF STANDARD ERRORS AND CHARTS

Exhibits 2 and 3 consist of both tables and charts. The tables show standard errors for specific numerical estimates. Interpolation for other values may be obtained by use of the accompanying charts. The following code identifies the appropriate exhibit to be used in estimating the standard error of the statistic described. The four components of each code describe the statistic as follows: (1) A = aggregate, P = percentage; (2) the number of calendar quarters of data collection; (3) the type of the statistic; and (4) the range of the statistic.

Exhibit 1. Guide to Use of Standard Errors and Charts

Statistic	*Use* Code	*Use* Exhibit
Number of:		
Persons in the U.S. population, or total number in any age-sex category ...	Not subject to sampling error	
Persons in any other population group	A4AN	2
Acute conditions: per year	A4BN	2
Persons with limitation of activity	A4AN	2
Persons injured	A4BN	2
Persons with hospital episodes	A4AN	2
Physician visits	A4BM	2
Disability days: per year	A4BW	2
Rates per 100 persons:		
Acute conditions: Per year	A4BN	2
Persons injured	A4BN	2
Days per person with episodes per year	Numer.: A4AW	2
	Denom.: A4AN	2
Disability days: per year	A4BW	2
Percentage distribution of:		
Persons with limitations of activity ...	P4AN-M	3
Persons with hospital episodes	P4AN-M	3
Persons by interval since last physician visit	P4AN-M	3
Physician visits per person per year	A4BM	2

Source: Adapted from: U.S. Department of Health, Education, and Welfare, *Current Estimates from the Health Interview Survey: United States–1967*, National Center for Health Statistics, Series 10, No. 52, pp. 35–36, 40.

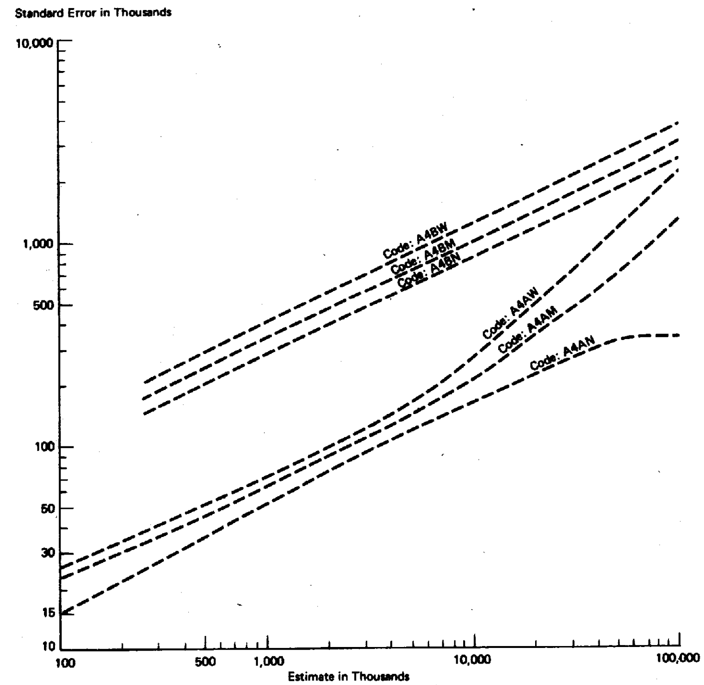
Exhibit 2.a. Standard Errors for Aggregates Based on Four Quarters of Data Collection for Data of All Types and Ranges

Size of estimate (thousands)	*Standard error (thousands)* Code A4AN	A4AM	A4AW	A4BN	A4BM	A4BW
100	15	22	25	—	—	—
250	25	32	38	140	170	200
500	35	45	50	200	240	290
1,000	52	62	70	280	340	400
2,000	72	88	96	380	460	560
5,000	120	140	160	600	700	880
10,000	160	210	260	820	1,000	1,200
20,000	220	340	480	1,200	1,400	1,700
30,000	260	460	700	1,400	1,700	2,100
50,000	300	700	1,200	1,800	2,200	2,700
100,000	340	1,300	2,200	2,500	3,100	3,800

NOTE: For interpolation refer to Exhibit 2.b.

Source: Adapted from U.S. Department of Health, Education, and Welfare, *Current Estimates from the Health Interview Survey: United States–1967*, National Center for Health Statistics, Series 10, No. 52, pp. 35–36, 40.

Exhibit 2b. Standard Errors for Aggregates Based on Four Quarters of Data Collection for Data of All Types and Ranges.



Source: Adapted from: U.S. Department of Health, Education, and Welfare, *Current Estimates from the Health Interview Survey: United States-1967*, National Center for Health Statistics, Series 10, No. 52, pp.35–36, 40.

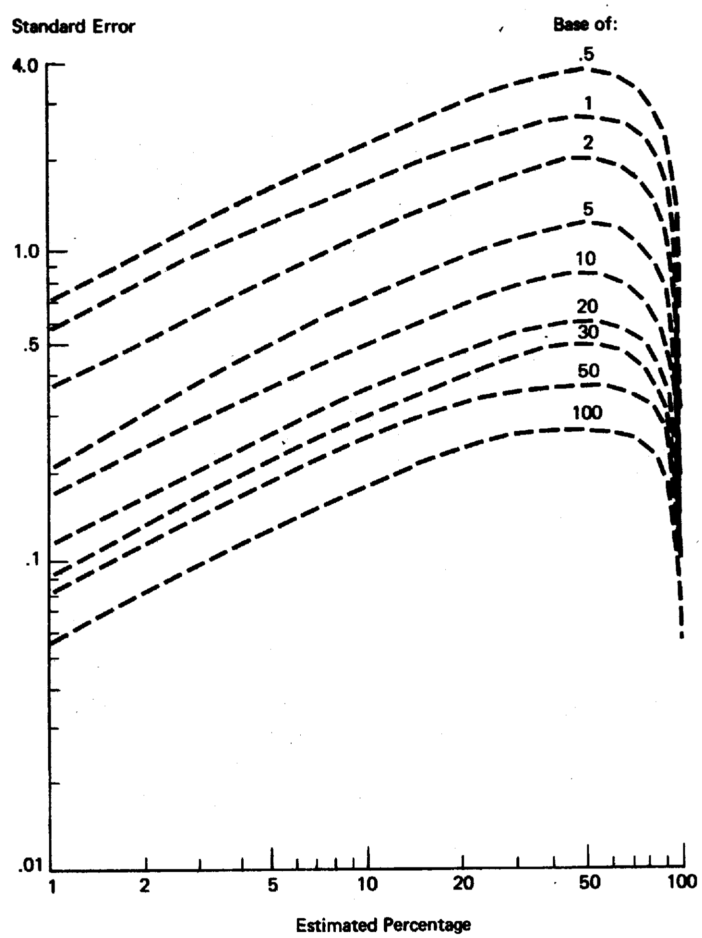
Exhibit 3.a. Standard Errors for Percentages Based on Four Quarters of Data Collection for Type A Data, Narrow and Medium Range
(Base in thousands)

Estimated percentages	*Base (thousands)*								
	500	*1,000*	*2,000*	*5,000*	*10,000*	*20,000*	*30,000*	*50,000*	*100,000*
1 or 99	0.7	0.5	0.4	0.2	0.2	0.1	0.1	0.1	0.1
2 or 98	1.0	0.8	0.5	0.3	0.2	0.2	0.1	0.1	0.1
5 or 95	1.6	1.2	0.8	0.5	0.4	0.3	0.2	0.2	0.1
10 or 90	2.2	1.6	1.1	0.7	0.5	0.3	0.3	0.2	0.2
15 or 85	2.6	1.9	1.3	0.8	0.6	0.4	0.3	0.3	0.2
25 or 75	3.2	2.3	1.6	1.0	0.7	0.5	0.4	0.3	0.2
35 or 65	3.5	2.5	1.8	1.1	0.8	0.5	0.5	0.3	0.2
50	3.7	2.6	1.9	1.2	0.8	0.6	0.5	0.3	0.3

NOTE: For interpolation refer to Exhibit 3.b.

Source: Adapted from U.S. Department of Health, Education, and Welfare, *Current Estimates from the Health Interview Survey: United States–1967*, National Center for Health Statistics, Series 10, No. 52, pp. 35–36, 40.

Exhibit 3b. Standard Errors for Percentages Based on Four Quarters of Data Collection for Type A Data, Narrow and Medium Range
(Base of percentage shown on curves in millions)



Source: Adapted from: U.S. Department of Health, Education, and Welfare, *Current Estimates from the Health Interview Survey: United States-1967*, National Center for Health Statistics, Series 10, No. 52, pp.35–36, 40.

APPENDIX C: STANDARD ERRORS OF ESTIMATED RATIOS AND OTHER STATISTICS THAT MAY BE DERIVED FROM THE PUBLISHED ESTIMATES

This appendix is an illustration of text describing how the published standard errors may be used to calculate approximate standard errors for selected types of derivative statistics.

The relative standard errors shown in the tables,[15] supplemented by the correlation coefficients given in Table 5′, may be used to develop approximate standard errors for various estimated ratios, $R' = Y'/X'$, estimated differences between such ratios and other measures that can be derived from the published estimates. Methods for computing such approximate standard errors are outlined next for estimated ratios and for differences between ratios.

C.1 Relative Standard Errors of Ratios

Approximate relative standard errors of ratios of different items for a given classification may be computed by

$$V(R') \doteq [V^2(Y') - 2\rho(Y',X')V(Y')V(X') + V^2(X')]^{\frac{1}{2}}, \quad \text{(C.1)}$$

where $V(Y')$ and $V(X')$ are the relative standard errors of each of the two item totals; $V^2(Y')$ and $V^2(X')$ are the squares of those relative standard errors; and $\rho(Y',X')$ is the correlation coefficient of the two estimates.

As an illustration, for Industry Group 201, Meat Products, the 1960 estimates of total employment and value added by manufacture are 311,000 employees and $2,742 million (Table 9). The estimated "value added per employee" ratio, therefore, is $8,800 per employee. Applying the method given in the footnote to Example 21, the relative error of the estimated value added total is about 0.02. As indicated in that footnote, the corresponding figure for the employment estimate for the same year is roughly the same.

Substituting 0.02 for each of $V(Y')$ and $V(X')$ and 0.8 for $\rho(Y',X')$, the mid-range value for Table 5′, gives

$$\begin{aligned} V(R') &\doteq [.0004 - 2(.8)(.02)(.02) + .0004]^{\frac{1}{2}} \\ &\doteq (.0001)^{\frac{1}{2}} \\ &\doteq .01, \end{aligned} \quad \text{(C.2)}$$

so that the estimated ratio of $8,800 per employee is subject to a relative standard error of about one percent.

[15] Table 9 in Example 21 is one of these tables; the other tables are not included here—Ed.

5′. Correlation Coefficients for Pairs of Different General Statistics Totals, Same Year and One Year Apart

Pairs of different general statistics totals	Range of correlation coefficients[a] Same year	One year apart
All, excluding inventories and capital expenditures	.75–.85	.65–.75
Inventories and any other except capital expenditures	.20–.50	.10–.30
Capital expenditures and any other	.00–.10	.00–.10

[a] Where industrial and geographic classifications are the same. Where classifications are different, correlation coefficients are zero.

The same general procedure may be applied to compute approximate relative standard errors of ratios involving different industrial or geographic classifications using the simpler formula

$$V(R') \doteq [V^2(Y') + V^2(X')]^{\frac{1}{2}}. \quad (C.3)$$

C.2 Standard Errors of Differences Between Ratios

Absolute standard errors of the estimated year-to-year change in a ratio for a given classification, $CR_{21}' = (Y_2'/X_2') - (Y_1'/X_1')$, may be computed, approximately, by using

$$S^2(CR_{21}') \doteq \left(\frac{Y_2' + Y_1'}{X_2' + X_1'}\right)^2 \{V^2(Y_2'/Y_1') + V^2(X_2'/X_1') - 2[\rho(Y_2',X_2')V(Y_2')V(X_2') + \rho(Y_1',X_1')V(Y_1')V(X_1') - \rho(Y_2',X_1')V(Y_2')V(X_1') - \rho(Y_1',X_2')V(Y_1')V(X_2')]\} \quad (C.4)$$

and then taking the square root of the result. In (C.4), Y_2', Y_1', X_2' and X_1' are the respective estimated totals; $V^2(Y_2'/Y_1')$ and $V^2(X_2'/X_1')$ are the estimated relative variances of the respective year-to-year ratios; and the other elements are estimates of the correlation coefficients and relative standard errors of the estimated totals. The relative variances of the year-to-year ratios may be obtained by squaring the corresponding relative standard errors given in the tables. The correlation coefficients $\rho(Y_2',X_2')$ and $\rho(Y_1',X_1')$ are approximately equal, as are $\rho(Y_2',X_1')$ and $\rho(Y_1',X_2')$, and may be obtained from Table 5′. The relative standard errors of the estimated totals $V(Y_2')$, etc., can be computed as indicated in the footnote to Table 9, Example 21.

Again using the data for Industry Group 201, Meat Products, as an illustration, the estimated value added per employee ratio increased from $8,400 to $8,800, or $400 per employee between 1959 and 1960. Inserting in (C.4) the two annual value added and employment estimates, the relative standard errors of the year-to-year changes for those items, the relative standard errors of the estimated totals (from Table 1[16] of the 1960 and 1961 *Annual Survey of Manufactures* volumes) and the mid-range values of the correlation coefficients (from Table 5′) gives

$$S^2(CR_{21}') \doteq \left(\frac{2{,}685 \times 10^3}{312}\right)^2 \{(.02)^2 + (.01)^2 - 2[(.80)(.022)(.014) + (.80)(.010)(.010) - (.65)(.022)(.010) - (.65)(.014)(.010)]\}. \quad (C.5)$$

Simplifying and taking the square root,

$$S(CR_{21}') \doteq \frac{2{,}685 \times 10^3}{312}[.0005 - 2(.00033 - .00023)]^{\frac{1}{2}}$$

$$\doteq 8{,}600(.0003)^{\frac{1}{2}} \quad (C.6)$$

$$\doteq 150 \text{ dollars per employee.}$$

The estimated increase of $400 value added per employee, therefore, is subject to a standard error of roughly $150 per employee.

An abbreviated, rougher computation of $S(CR_{21}')$ may be made by omitting the correlation terms. The resulting simplified formula

$$S(CR_{21}') = \left(\frac{Y_2' + Y_1'}{X_2' + X_1'}\right)[V^2(Y_2'/Y_1') + V^2(X_2'/X_1')]^{\frac{1}{2}}, \quad (C.7)$$

will generally overstate the value given by the more complete formula, (C.4). For example, for the value added per employee ratio given above, the simple formula gives $S(CR_{21}') \doteq \$190$ value added per employee, compared with the result $S(CR_{21}') \doteq \$150$ value added per employee from the more complete formula.

[*Received March 1974. Revised February 1975.*]

[16] Table 1 not included here—Ed.

SELECTED BIBLIOGRAPHY

1. Bureau of the Budget, Executive Office of the President, *Statistical Services of the U.S. Government*, rev. ed., Washington, D.C.: U.S. Government Printing Office, 1968.
2. Office of Management and Budget, Executive Office of the President, *Standards and Guidelines for Federal Statistics*, Circular A-46, Washington, D.C.: U.S. Government Printing Office, 1974 (revision).
3. President's Commission on Federal Statistics, *Federal Statistics*, Vols. I and II, Washington, D.C.: U.S. Government Printing Office, 1971.
4. Sirken, Monroe G., *et al.*, "A Manual for Standards and Procedures for Reviewing Statistical Reports," National Center for Health Statistics, U.S. Department of Health, Education, and Welfare, 1974 (unpublished).
5. Statistical Office of the United Nations, "The Preparation of Sampling Survey Reports," *The American Statistician*, 3 (June–July 1950), 6–10.
6. Statistical Office of the United Nations, *Recommendations for the Preparation of Sample Survey Reports (Provisional Issue), Statistical Papers*, Ser. C, No. 1, Rev. 2, New York, 1964.

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